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On multiple equilibria and the rational expectations hypothesis★

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Summary. This paper analyzes through a simple two-period model the fact that, if some agents hold inside money intertemporally, the second-period "normalization" matters. Thus, there are several equilibria of the second-period economy, indexed by the level of inflation. A concept of equilibrium acknowledging this fact, and requiring that agents put some weight on any of the possible second-period equilibrium price vectors is developed. Such an equilibrium is shown to exist, and is illustrated by an example.

1. Introduction

It is well-known that an equilibrium in an Arrow-Debreu economy with a full array of markets for contingent claims is also an equilibrium of the sequential economy with rational expectations and complete financial markets (see Arrow [1953]). This result allows one to reduce the study of an *a priori* dynamic economy to the one of a static model. In my opinion, this reduction violates the timing of market opening.

Loosely speaking the argument in a two-period economy goes as follows. If for a given allocation in the first period, there are several possible equilibria (in the static sense) in the second period, how can agents be sure, in a decentralized framework, that a particular equilibrium will arise? The rational expectations hypothesis gives the following answer. For a given price today there is at most one price vector tomorrow such that if everybody expects it, it clears today's and tomorrow's markets. Hence, agents should focus their expectations on this particular price vector, and, comes the second period, this will be the observed price. This argument has in my opinion the defect of relying on the fact that tomorrow's prices are announced today by some auctioneer. However, if one takes the timing of trading

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and of price formation into account, it seems more consistent to say that second-period prices should be decided by the second-period auctioneer on the basis of the data of the economy at that point in time. Then, the rational expectations equilibrium second-period price is just one among many.

Therefore, the rational expectations approach relies on either assuming that tomorrow's prices are quoted today, which goes against the dynamic nature of the model, or that the second-period auctioneer will systematically choose, among many others, one particular price vector, which is a very peculiar and unexplained price selection.

The above observations suggest that, in the absence of a mechanism that picks candidate equilibrium out of the many possible equilibria of the second-period economy, agents should place some positive weight on any of them, and hence, instead of having point expectations, have what I will call rational set expectations. It is defined as follows. Agents are required to have a probability distribution about future prices whose support is the set of possible spot equilibrium prices in the continuation economy, given the first-period allocation. Compared to rational expectations, this concept has the advantage of respecting the sequentiality of trading without imposing an equilibrium selection mechanism. Compared to standard temporary equilibrium models (see Grandmont [1977]), it restricts the set of allowable expectation patterns, and cannot be disregarded as *ad hoc* since it is based on agents' "rationality" and equilibrium conditions. It bears some resemblance with the concept of rationalizable equilibrium in game theory (see Pearce [1984] and Bernheim [1984]). It is also somehow related to the notion of endogenous uncertainty (see *e.g.* Chichilnisky and Wu [1991]), which says that the model "generates" uncertainty because of the possible multiplicity of second-period equilibrium.

This relates to a similar discussion in Drèze [1993]. There, he distinguishes among the two approaches to dynamic general equilibrium theory, namely the rational expectations-incomplete market literature and the temporary equilibrium literature, in the following way:

"Either states are described so comprehensively that conditional prices are well defined, but then states are not objectively observable; or states are objectively observable, but conditional prices cannot be taken as known." Drèze [1993] p. 4.

He then reinterprets the incomplete market literature in the following way[1]:

"One possible interpretation [of this literature] would be that there exists a set of events defined with operational objectivity; given any such event, there exists a set of alternative states, not distinguishable with operational objectivity, but defined *conceptually* with enough detail that a unique equilibrium is associated with each state. *This comes close to associating with each event a set of alternative equilibria, over which individuals are allowed to hold idiosyncratic expectations.*" Drèze [1993] p. 5.

What is done in this paper is precisely to try to see what a model corresponding to the last sentence in the above quotation would look like.

The interest of the equilibrium concept lies in the fact that it allows for some uncertainty at each period about the equilibrium allocation. Indeed, in standard

[1] The italics are by the author.

rational expectations representations, the equilibrium path is set once and for all at the beginning of time. Here, the equilibrium allocation depends, at each period on which (static) equilibrium price is selected. Thus, this equilibrium concept allows for a representation of the economy in which agents do not make mistakes (they always place a positive probability on the price that will actually be observed) but in which the equilibrium allocation in each period is not predetermined (in the sense that it is not known at the beginning of time).

Section 2 contains the set-up of a two-period general equilibrium model and some well-known properties of rational expectations equilibria. I prove in section 3 that, typically, the equilibrium in the continuation economy is indeterminate. This leads me to define and discuss a new concept of equilibrium in section 4. This section also contains a sufficient assumption to prove existence, as well as an example. Section 5 is devoted to a discussion of the results obtained and to concluding remarks.

2. The model

In this section, I lay down a simple model to show that rational expectations are somehow not "time consistent"[2]. There are two periods, $t = 0, 1$, with no uncertainty in the second. This is a simplification that is of no great loss of generality in the present framework. It is equivalent to saying, in Drèze's terminology, that only one state can be defined with operational objectivity. H households, labelled by the subscript h, trade C commodities ($c = 1, \ldots, C$) at each date, and can transfer wealth from one period to another through trade in inside money. Commodity c consumed by household h at time t will be denoted $x_h^{t,c}$, with $x_h^t = (x_h^{t,c})_{c=1}^C$, $x_h = (x_h^0, x_h^1)$ and $x = (x_h)_{h=1}^H$. Associated prices are $p = (p^0, p^1)$. Holdings of inside money is denoted m_h, and its price is q. Each household has a utility function u_h and endowments e_h, on which I make the following assumption.

Assumption 1: *$u_h: \mathbb{R}_+^{2C} \to \mathbb{R}$ is smooth, differentiably strictly increasing, differentiably strictly concave, bounded above and has indifference curves closed in $\mathbb{R}_{++}^{2C}$. Endowments are strictly positive.*

In the rest of the paper I parameterize the economy by the endowment vector e. Under the rational expectations hypothesis, household h solves the following problem:

$$\max_{x_h^0, x_h^1, m_h} u_h(x_h^0, x_h^1) \quad \text{s.t.} \quad \begin{cases} p^0(x_h^0 - e_h^0) = -qm_h \\ p^1(x_h^1 - e_h^1) = m_h \\ \quad (x_h^0, x_h^1) \in \mathbb{R}_+^{2C} \end{cases} \qquad (P)$$

A rational expectations equilibrium is then a price vector $(p, q) \in \mathbb{R}_{++}^{2C} \times \mathbb{R}_+$ and an allocation $(x, m) \in \mathbb{R}_{++}^{2CH} \times \mathbb{R}^H$ such that, given (p, q), (x_h, m_h) solves (P) for all h, and markets clear, *i.e.* $\sum_h x_h = \sum_h e_h$ and $\sum_h m_h = 0$.

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[2] This could be related to what Guesnerie [1993] calls the "uniqueness viewpoint" according to which rational expectations would make sense only if the second-period spot equilibrium is unique.

Under the above assumption, it is well known (see, *e.g.* Debreu [1970]) that for an open and dense set of endowments (the set of regular economies) equilibria are locally unique.

Furthermore, it is easy to show that, given utility functions, for an open and dense set of regular economies denoted X, some household will transfer wealth from a period to the other, at equilibrium. In other terms, there is a household h such that $m_h \neq 0$. This property will be useful to show that there is typically a continuum of equilibria in the continuation economy (the one made of second-period data).

3. Indeterminacy of equilibrium in the continuation economy

Let now study the continuation economy and fix $e \in X$. Let (x^0, m) be arbitrary actions taken in the first period. Markets then reopen and an equilibrium price of this economy is a price such that markets clear at this time, given the actions (x^0, m). At this point, one has to acknowledge the fact that households come to the second period with non-zero monetary balances. Hence, it is possible that, for some prices, they will not be able to reimburse their debt. Denoting by K the fraction of what creditors get of what they are owed ($K \in [0,1]$), and letting $m_h^+ = \max(0, m_h)$ and $m_h^- = -\min(0, m_h)$, the problem to be solved can be written:

$$\max_{x_h^1} u_h(x_h^0, x_h^1) \quad \text{s.t.} \quad \begin{cases} p^1(x_h^1 - e_h^1) = Km_h^+ - \min(Km_h^+ + p^1 e_h^1, m_h^-) \\ x_h^1 \in \mathbb{R}_+^C \end{cases} \qquad (P_1)$$

Then, one could define an equilibrium concept at which bankruptcy is allowed (*i.e.* such that $K < 1$). However, for the sake of simplicity, I will define $\bar{p}^1$ to be an equilibrium of the continuation economy if, given this price, households solve (P_1), $\min(m_h^+ + \bar{p}^1 e_h^1, m_h^-) = m_h^-$ for all h, and markets clear. At such a price vector no default occurs (*i.e.* $K = 1$). More formally, let z^1 be the excess demand for goods in period 1 and define the equilibrium price set of the continuation economy without bankruptcy to be:

$$E^1(x^0, m) = \left\{ p^1 \in \mathbb{R}_{++}^C \,\middle|\, \sum_h z_h^1(x_h^0, m_h, p^1) = 0 \text{ and } \min(m_h^+ + p^1 e_h^1, m_h^-) = m_h^- \text{ for all } h \right\}$$

Clearly, a necessary condition for E^1 to be non-empty is that $\sum_h m_h = 0$. It is also intuitive that the equilibrium allocation associated with different prices are different. This can easily be seen in the one good case. Consumption in period one is equal to $e_h^1 + m_h/p^1$, which is strictly positive for high enough p^1. Then, an equilibrium exists if the money market cleared in period 0. Furthermore, the equilibrium allocation varies with prices if there is a household with non-zero money balances. Observe that in this case, the no-default condition imposes that equilibrium prices are in the interval $(\max_h(-m_h/e_h^1), +\infty)$. In fact, any price in the interval is an equilibrium price if $\sum_h m_h = 0$. The no-default condition implies that prices can not be too low in the second period, or in other terms, inflation is high enough to allow debtors to be solvent.

Proposition 1 states formally the property that "inflation matters" in the continuation economy. Let

$$X^1(x^0, m) = \{x^1 \in \mathbb{R}_+^{CH} \mid \exists p^1 \in E^1(x^0, m) \text{ s.th. } x_h^1 = x_h^1(x_h^0, m_h, p^1) \text{ for all } h\}$$

be the set of equilibrium allocations.

Proposition 1. *Let m be such that* $\sum_h m_h = 0$ *and* $m_h \neq 0$ *for some h. Then,* $X^1(x^0, m)$ *contains a continuum of points.*

Proof: Let (x^0, m) be a first-period allocation such that $\sum_h m_h = 0$ and $m_h \neq 0$ for some h. Let $v \in \mathbb{R}_+$ be such that $e_h^1 + (m_h/v) \gg 0$ where, with a slight abuse of notation m_h denotes the C-dimensional vector $(m_h, \ldots, m_h)$. Observe that such a parameter exists since $e_h^1 \gg 0$ for all h. Then, we know there is an equilibrium price vector in the continuation economy such that $\sum_c p^{1,c} = v$. Indeed, it is now equivalent to a standard static economy with endowments equal to $e_h^1 + (m_h/v)$ for which we know there is an equilibrium. Besides, since $\sum_h m_h = 0$ the equilibrium allocation is such that $\sum_h x_h^1 = \sum_h e_h^1$. Moreover, it is possible to show that the equilibrium allocations depends on v. Let $V = (\max_{c,h}(-m_h/e_h^{1,c}), +\infty)$ and define γ by:

$$\gamma : V \to \mathbb{R}^C_{++}$$
$$v \mapsto \gamma(v) = (p^{1,2}, \ldots, p^{1,C}) \text{ s.th. } z^1\left(x^0, m, v - \sum_c \gamma^c(v), \gamma(v)\right) = 0$$

Define $\mu(v) = (x_h^1(x_h^0, m_h, v - \sum_c \gamma^c(v), \gamma(v)))$, and observe that $\mu(V) \subset X^1(x^0, m)$. Observe that μ is one to one: let $v_1 \neq v_2$ and suppose that $\mu(v_1) = \mu(v_2)$. From the first order conditions, this gives $D_{x^1} u_h(x_h^0, \mu(v_1)) = D_{x^1} u_h(x_h^0, \mu(v_2))$, *i.e.*, $(v_1, \gamma(v_1)) = \alpha(v_2, \gamma(v_2))$ for some α. But the budget constraint then yields:

$$(v_1, \gamma(v_1))(\mu(v_1) - e_h^1) = -p^0(x_h^0 - e_h^0) = (v_2, \gamma(v_2))(\mu(v_2) - e_h^1)$$

which implies $\alpha = 1$ if $p^0(x_h^0 - e_h^0) \neq 0$, a contradiction. □

Thus, if one takes the timing of the decisions and trades into account (and especially that markets reopen at the beginning of the second period) and observes that the second-period outcome is indeterminate viewed from the end of the first period, it is difficult to see how agents could "coordinate" their expectations to come up with the first period component of a rational expectations equilibrium to begin with.

Even though the argument is cast in a very simple model, it should be clear that it only relies upon the non-homogeneity in prices of the second-period budget constraints, which is more general than the model. However, it could be noticed that even if these budget constraints were linear homogeneous in prices (*i.e.*, if we had real assets, or if money holdings were zero), there is still one degree of nominal indeterminacy, even though the allocation reached at equilibrium is the same at all levels of inflation. Then, a problem remains since it is not clear how agents could come up with the same expectations in period 0, an assumption needed to observe the first-period component of any rational expectations equilibrium.

4. Rational set expectations

The above discussion shows that, typically, the continuation equilibrium in a rational point expectations equilibrium is indeterminate. This suggests a way to get around this difficulty by defining "consistent" set expectations and lead to a straightforward change in the definition of what an equilibrium could be in this type of models. On top of market clearing, one has to impose that, in equilibrium, agents forecast accurately and, loosely speaking, put some positive weight on almost every possible equilibrium outcome in the second period, given the equilibrium allocation observed today. Such an equilibrium will be called a rational set expectations equilibrium.

Thus, for any (x^0, m) and any fixed subjective probability distribution Ψ_h over future spot prices p^1, define agent h's preferences over current actions as in standard temporary equilibrium models, by the indirect utility function:

$$v_h(x_h^0, m_h, \Psi_h) = \int u_h(x_h^0, x_h^1(x_h^0, m_h, \cdot))d\Psi_h,$$

where $x_h^1(x_h^0, m_h, p^1)$ is a solution to (P_1). Such an utility function exists since u_h is bounded.

One can now state the definition of a L-rational set expectations equilibrium.

Definition: A L-rational set expectations equilibrium is a price system $(\bar{p}^0, \bar{q})$ and an allocation $(\bar{x}^0, \bar{m})$ together with subjective probability distributions Ψ_h such that

(*i*) $(\bar{x}_h^0, \bar{m}_h)$ maximizes $v_h(x_h^0, m_h, \Psi_h)$ under $\bar{p}^0(x_h^0 - e_h^0) + \bar{q}m_h = 0$ and $|m_h| \leq L$,
(*ii*) markets clear, *i.e.*, $\sum_h(\bar{x}_h^0 - e_h^0) = 0$, and $\sum_h \bar{m}_h = 0$,
(*iii*) the support of each subjective probability distribution Ψ_h coincides with the equilibrium price set of the continuation economy, *i.e.*, Supp $\Psi_h = E^1(\bar{x}^0, \bar{m})$.

Hence, a L-rational set expectations equilibrium is a price vector and an allocation today (with money holdings bounded by L) such that there exist probability distributions such that when agents maximize their expected utility (computed according to each agent's distribution function) given that they expect (almost) all the possible equilibria in the (static) continuation economy, markets clear today. The only coordination that is now required on agents' expectations is that they are equilibrium expectations[3]. This concerns only the support of the probability measure and not its shape. It is assumed here, without loss of generality, that the bound on money holdings is the same for all agents. This bound, is imposed, in the spirit of Radner [1972], to prove existence of an equilibrium.

Observe that, the budget constraint being linear homogeneous in (p^0, q), multiplying $(\bar{p}^0, \bar{q})$ by a positive constant yields the same RSEE. It is also the case that multiplying $(\bar{p}^0, \bar{m}, \Psi_h)$ by a positive number (such that the new money holdings are less than L) generates another RSEE with the same real allocation x^0.

A L-rational set expectations equilibrium is generally different from the first-period allocation of a rational point expectations equilibrium and hence does not possess all the optimal properties of the latter. This is not surprising since the implicit selection mechanism used by the second-period auctioneer consists of picking the equilibrium price in the second period such that the two-period allocation then reached is Pareto optimal.

The following assumption on individuals' probability distributions is sufficient to prove existence.

Assumption 2: *Let $M(\mathbb{R}_{++}^C)$ be the set of probability measures on $\mathbb{R}_{++}^C$, endowed with the weak convergence topology. Then,*

[3] There is actually a small caveat to this statement: agents expect equilibria at which no default occurs.

(*i*) *there exists* (*at least*) *one continuous function* Φ *from* $\mathbb{R}^{CH}_{++} \times \mathbb{R}^{H}$ *to* $M(\mathbb{R}^{C}_{++})$ *such that* $\text{Supp } \Phi(x^0, m) = E^1(x^0, m)$ *if* $\sum_h x^0_h = \sum_h e^0_h$ *and* $\sum_h m_h = 0$, *and*
(*ii*) *the correspondence* $\sigma: (x^0, m) \mapsto \text{Supp } \Phi(x^0, m)$ *is upper hemi-continuous.*

This assumption states the existence of a continuous function with the property that when markets clear in the first period, its support (*i.e.*, all the expected prices) consists of all the second-period equilibrium prices. The functions Φ_h need not be interpreted as actual expectations function since an agent's information at date 0 is (p^0, q) and not the entire allocation.

This assumption places some restrictions on the correspondence σ even for allocations that do not clear markets in the first period. It also has the implication that σ is lower hemi-continuous (Green [1973]). Indeed, let p be in $\sigma(x^0, m)$. Then, if G is an open neighborhood of p, $\Phi(x^0, m)(G) > 0$, since $\sigma(x^0, m) = \text{Supp } \Phi(x^0, m)$. Let (x^0_k, m_k) be a sequence converging to (x^0, m). If σ were not l.h.c., there would exist an open neighborhood F of p such that F and $\sigma(x^0_k, m_k)$ are disjoint for infinitely many k. Hence, $\Phi(x^0_k, m_k)(F) = 0$ for infinitely many k. But weak continuity of Φ is equivalent to $\liminf \Phi(x^0_k, m_k)(G) \geq \Phi(x^0, m)(G)$ for all G open. Thus $\Phi(x^0, m)(G) = 0$, a contradiction.

In particular, this assumption implies that $E^1(.,.)$ is continuous on the set of points (x^0, m) such that $\sum x^0_h = \sum e^0_h$ and $\sum m_h = 0$. This shows that it is not totally innocuous. An example shows its precise meaning and that it can be satisfied.

Theorem 1: *Under assumptions 1 and 2, there exists a L-rational set expectations equilibrium.*

The proof is in the appendix.

Thus, if functions Φ_h satisfying assumption 2 exist, there is a temporary general equilibrium, which is then a rational set expectations equilibrium.

Notice that the equilibrium reached might depend on the bound on money holding. One could then define a rational set expectations equilibrium to be the limit of a sequence of *L*-rational set expectations equilibrium (or, directly, to be such that agents do not face any constraint other than the budget and the no-default constraints). The problem with existence of such an equilibrium is that if agents expect an infinite rate of inflation, they will hold (negative) infinite amount of money, whose price will go to zero. It is then possible that no equilibrium exists.

To give some intuition about this equilibrium concept, I now present an example where it is possible to compute a rational set expectations equilibrium.

There are two agents and one good. The first agent's utility is $u_1 = (x^0_1)^{1/2}(x^1_1)^{1/2}$ and his endowment is equal to $(1, 0)$. The second agent's preferences can be represented by the utility function $u_2 = \frac{1}{2}\log x^0_2 + x^1_2$, and his endowment is $(0, 1)$.

It is easy to compute demand functions under the rational expectations hypothesis. Let $q = 1$, one has:

$$x^0_1 = 1/2, \quad x^1_1 = p^0/2p^1, \quad m_1 = p^0/2$$

and

$$x^0_2 = p^1/2p^0, \quad x^1_2 = 1/2, \quad m_2 = -p^1/2$$

Thus, the equilibrium prices are such that $p^0/p^1 = 1$ (there is still one normalization that could be done).

Let now study the rational set expectations equilibrium (I assume there are no bounds on money holdings). Let Φ be the subjective probability distribution of the first agent on future equilibrium prices. Denote its support by $[\underline{p}, +\infty)$, where $\underline{p}$ is going to be determined through the no-default constraint. Similarly, let Ψ be the subjective probability distribution of the second agent, which has the same support.

After replacing x_1^0 and x_1^1 by their expression in terms of m_1 (obtained from the budget constraints) in the utility function, the problem 1 has to solve is the following:

$$\max_{m_1} \int_{\underline{p}}^{+\infty} \left(1 - \frac{m_1}{p^0}\right)^{1/2} \left(\frac{m_1}{p^1}\right)^{1/2} \mathrm{d}\Phi$$

which yields:

$$m_1 = \frac{p^0}{2}.$$

Similarly, 2 has to solve (m_2 is negative):

$$\max_{m_2} \int_{\underline{p}}^{+\infty} \left(\frac{1}{2}\log\left(-\frac{m_2}{p^0}\right) + \left(1 + \frac{m_2}{p^1}\right)\right)\mathrm{d}\Psi$$

which yields:

$$m_2 = -\frac{1}{2\hat{p}^1} \quad \text{where} \quad \hat{p}^1 = \int_{\underline{p}}^{+\infty} \frac{1}{p^1}\mathrm{d}\Psi.$$

Thus, the candidate equilibrium price is $\bar{p}^0 = 1/\hat{p}^1$.

The last things to find are the probability distributions and their support. First, notice that at the candidate equilibrium price, the money market clears in the first period and thus $x_1^1 + x_2^1 = 1$ for all positive p^1. Second, observe that since $m_1 > 0$ there is no bankruptcy problem for agent 1, and Φ could be any probability distribution on $[\underline{p}, +\infty)$ (such that $\tilde{p}^1 = \int_{\underline{p}}^{+\infty} (p^1)^{-1/2}\mathrm{d}\Phi$ exists, *i.e.* the integral converges).

Now, let study the no-default condition for agent 2. The following inequality must be satisfied, in order for agent 2 to be solvent in the second period:

$$1 + \frac{-1/2\hat{p}^1}{p^1} \geq 0 \quad \text{for all} \quad p^1 \geq \underline{p}$$

or

$$2p^1 \geq \frac{1}{\int_{\underline{p}}^{+\infty} \frac{1}{p^1}\mathrm{d}\Psi} \quad \text{for all} \quad p^1 \geq \underline{p}$$

I show that there exists a density ψ such that the following holds:

$$2\underline{p} = \frac{1}{\int_{\underline{p}}^{+\infty} \frac{1}{p^1}\psi(p^1)\mathrm{d}p^1} \quad \text{and} \quad \int_{\underline{p}}^{+\infty} \psi(p^1)\mathrm{d}\underline{p}^1 = 1$$

and thus, that assumption 2 is satisfied. Let $\psi(p) = p^{\alpha}$, with $\alpha < -1$. Then, it is possible to show that there exist α and $\bar{p}$ such that the system above holds (more specifically, $\alpha = -2$, and $\bar{p} = 1$). One then gets that $\hat{p}^1 = 1/2$ and $\bar{p}^0 = 2$.

Thus, the conclusion is that if the first agent has any expectations whose support is $[1, +\infty)$, and the second agent has expectations as defined above, then there is a rational set expectations equilibrium given by $\bar{p}^0 = (1/2\hat{p}^1) = 2$. There are of course other possible functions ψ, which would yield different $\bar{p}$, and equilibrium prices p^0. However, the purpose of the example is only to show how one can go on finding an equilibrium. Furthermore, it shows that, in that particular case, it is fairly easy to find probability distributions that satisfy assumption 2.

5. Discussion and concluding remarks

A *L*-rational set expectations equilibrium has been shown to exist under assumption 2, which states the existence of continuous equilibrium expectations. The next natural step in the analysis would be to find a class of economies for which that assumption is satisfied, as it is the case, for example, if $E^1(x^0, m) = E^1(\sum_h e_h^0, 0)$. What makes it hard is the "ambiguous" status of expectations in this model. They are endogenous, being equilibrium expectations, but this requirement is not enough to fully characterize them, leaving room for some exogenous assumptions. In temporary equilibrium, expectations are given exogenously. On the other hand, no equilibrium condition is imposed in the second period. On the contrary, in the rational expectations approach, markets clear in the second period and expectations are fully endogenized, and have the same exact status as current prices. What I have argued in this paper is that the market clearing conditions in the (static) continuation economy are not enough to pinpoint a particular equilibrium price expectation. But when one gets away from point expectations, one need to specify a probability measure according to which expectations are distributed. Since there is no modelled equilibrium selection mechanism in this economy, the distribution reflects one's own view about how markets work, and there is no further endogenous restrictions on these probability measures.

One could also note that the present analysis is in spirit similar to the concept of rationalizable equilibrium developed in game theory (see Bernheim [1984] and Pearce [1984]). The question asked there is "are there any restrictions of individuals' expectations which are required by rationality alone rather than by (subjective) plausibility?" (Bernheim [1984]). In the present competitive framework, expectations are not about other agents' expectations but rather about the market outcome itself, which is independent of any particular agent's actions. Hence, expectations are about how markets work. What I have been arguing then is that agent's rationality in itself does not imply much concerning their expectations about market's functioning.

Observe that if every household had the same probability distribution, then the model becomes one of a sunspot economy, where the sunspot acts as an equilibrium selection mechanism. The sunspot can take an uncountable number of values, each state being determined by a price normalization and, if necessary, an index denoting a particular equilibrium at a given price normalization. If such an equilibrium exists, then a RSEE exists. This would be an alternative approach to the existence problem[4]. In that view, one would assimilate a model with *a priori* no uncertainty to a model with uncountably many states, each state corresponding, loosely speaking to a particular value of money[5]. On a more general and informal level this means that it is difficult to define a decentralized two-period model in which uncertainty has no place.

Finally, it is clear that the issue is that of price formation in these competitive models. Indeed, how to model expectations crucially depends on what are the underlying "true" economic processes at work, and on whether they can be defined independently of how agents think they are operating.

Appendix

Proof of theorem 1: Before proving this result, observe that an equilibrium of the model where on top of the first-period budget constraint $p^0(x_h^0 - e_h^0) = -qm_h$ and $|m_h| \leq L$, one imposes that $p^1 e_h^1 + m_h \geq 0$ for all $p^1 \in \text{Supp}\, \Phi_h(x^0, m)$, is an equilibrium of the original model. This is the case since we imposed that there is no bankruptcy at the second-period equilibrium and agents expect only equilibrium prices. Let $c_h(x_0, m) = \min_{p^1 \in \sigma_h(x^0,m)} (p^1 e_h^1)$. c_h exists, is positive and is continuous since σ_h is continuous, $e_h^1 \gg 0$, and $\sigma_h(x^0, m)$ is a closed subset of $\mathbb{R}_+^C$.

Endow agents with functions Φ_h that satisfy assumption 2. Notice first that v_h is continuous in $(x_h^0, m_h, \Phi_h(x^0, m))$ (see Grandmont [1974]). The indirect utility function is monotone and strictly concave in (x_h^0, m_h). This follows from similar properties of u_h.

Since v_h is strictly increasing, a necessary condition for demand to be well-defined is that $p^0 \gg 0$. That this is sufficient is now established. Let

$$\beta_h(p^0, q, x^0, m) = \{(x_h^0, m_h) \in \mathbb{R}_+^C \times \mathbb{R} \,|\, p^0(x_h^0 - e_h^0) = -qm_h, |m_h| \leq L, \text{ and } m_h \geq -c_h(x^0, m)\}$$

be the budget correspondence. First, observe that it is closed. Now, since m_h is bounded, x_h^0 is bounded if $p^0 \gg 0$, and hence $\beta_h(p^0, q, x^0, m)$ is compact when $p^0 \gg 0$. Note also that β_h is convex-valued.

It is straightforward to see that β is u.h.c. To prove that it is l.h.c. as well, let $\hat{\beta}$ be the correspondence defined as β except for inequalities (including $x_h^0 \geq 0$) that become strict inequalities. Let (p_k^0, q_k, x_k^0, m_k) converge to (p^0, q, x^0, m) such that $p^0 \gg 0$. Note that $(e_h^0, 0)$ belongs to $\hat{\beta}(p^0, q, x^0, m)$ since $e_h^0 \gg 0$ and $c_h > 0$. Let $(x_h^0, m_h) \in \hat{\beta}(p^0, q, x^0, m)$. Define $m_{hk} = m_h - c_h(x^0, m) + c_h(x_k^0, m_k)$, and observe that it converges to m_h, and that $m_{hk} > c_h(x_k^0, m_k)$. Also, $|m_{hk}| < L$ for large enough k, since $|m_h| < L$. Define $x_{hk}^{0c} = (p^{0c} x_h^{0c} + \varepsilon_k / C) / p_k^{0c}$ where $\varepsilon_k = (p_k^0 - p^0) e_h^0 - q_k m_{hk} + q m_h$. Since $\varepsilon_k \to 0$, and $(x_h^0, p^0) \gg 0$, for sufficiently large k, $x_{hk}^0 \gg 0$. Also, $p_k^0 x_{hk}^0 = p_k^0 e_h^0 - q_k m_{hk}$ for all k. Hence there is a sequence in $\hat{\beta}(p_k^0, q_k, x_k^0, m_k)$ such that it converges to (x_h^0, m_h). Thus, $\hat{\beta}$ is l.h.c., and therefore β is l.h.c. as well, and hence continuous. A standard application of the maximum theorem then yields the existence and continuity of the demand function.

From the previous argument, one gets the boundary condition,

$$\| x_h^0(p_k^0, q_k, x_k^0, m_k, \Phi_{hk}(x_k^0, m_k)) \| \to +\infty$$

when p_k^0 goes to a vector with some zero components.

Let $\Delta_k = \{(p^0, q) \in \mathbb{R}_+^{C+1} \,|\, q \geq 1/k, p^{0c} \geq 1/k, c = 1, \ldots, C \text{ and } \sum_c p^{0c} + q = 1\}$, and $\Delta = \bigcup_k \Delta_k$. Δ_k is compact and convex. Define Ω to be the set of "concievable" allocations, *i.e.*

$$\Omega = \left\{ (x^0, m) \in \mathbb{R}_+^{CH} \times \mathbb{R}^H \,|\, |m_h| \leq L, 0 \leq x_h^0 \leq \sum_h e_h^0 \text{ for all } h \right\}$$

[4] Interestingly enough, this could be related to the existence of an equilibrium with a general state space, as in Mas-Colell and Monteiro [1991].

[5] This is exactly what Drèze [1993] argues: "In models incorporating a "positive theory of inflation", it would seem natural to describe events so comprenhensively that *nominal prices* are well defined."

Ω is compact and convex as well. Let $Z_k \subset \mathbb{R}^{CH} \times \mathbb{R}^H$ be a convex, compact set such that $(z^0, m)(\Delta_k \times \Omega) \subset Z_k$. That is $(z^0_h, m_h)(p^0, q, x^0, m, \Phi_h(x^0, m)) \in Z_k$ for all $(p^0, q) \in \Delta_k$ and $(x^0, m) \in \Omega$.

Let $\mu_k: Z_k \to \Delta_k$ be defined by:

$$\mu_k(z^0, m) = \left\{ (p^0, q) \in \Delta_k | (p^0, q) \text{ maximizes } p^0 \sum_h z^0_h + q \sum_h m_h \right\}.$$

μ_k is compact- and convex-valued, and it is u.h.c. Walras' law yields[6]

$$p^0 \sum_h z^0_h(p^0, q, z^0, m) + q \sum_h m_h(p^0, q, z^0, m) = 0$$

Define now γ_k as follows:

$$\gamma_k: \Delta_k \times Z_k \to \Delta_k \times Z_k$$
$$(p^0, q, (z^0_h, m_h)^H_{h=1}) \mapsto (\mu_k(z^0, m), ((z^0_h(p^0, q, z^0, m), m_h(p^0, q, z^0, m))^H_{h=1})$$

The domain is compact and convex, and given the continuity properties of the demand functions, the correspondence γ_k is compact- and convex-valued, and it is u.h.c.. Hence, Kakutani's fixed point theorem applies and γ_k has a fixed point. Now, consider the sequence of these fixed points as k goes to infinity. It must be the case that $(p^0_k, q_k) \to (\bar{p}^0, \bar{q})$.

One also has that $p^0 \sum_h z^0_{hk} + q \sum_h m_{hk} \leq 0$ for all $(p^0, q) \in \Delta_k$. This implies, since $\{z^0_{hk}\}$ is bounded below by $-e^0_h$ and $\{m_{hk}\}$ is bounded, that the sequence $\{z^0_{hk}, m_{hk}\}$ is bounded and thus converges to $(\bar{z}^0_h, \bar{m}_h)$. From the boundary condition, this yields that $\bar{p}^0 \gg 0$. Therefore, by continuity of demand, $(\bar{z}^0_h, \bar{m}_h)$ is the demand at $(\bar{p}^0, \bar{q})$.

Thus, one has:

$$\bar{p}^0 \sum_h \bar{z}^0_h + \bar{q} \sum_h \bar{m}_h = 0$$

and

$$p^0 \sum_h \bar{z}^0_h + q \sum_h \bar{m}_h \leq 0 \quad \text{for all } (p^0, q) \in \Delta$$

Suppose now that $\bar{q} = 0$. Then, the above system implies that $\sum_h \bar{z}^0_h = 0$, and hence that $q \sum_h \bar{m}_h \leq 0$ for all q. Thus, $\sum_h \bar{m}_h \leq 0$. However, if $\bar{q} = 0$, it is clear that the demand for money, $m_h(\bar{p}^0, \bar{q}, \bar{x}^0, \bar{m})$ is maximal and equal to L for all h. But then, $\sum_h \bar{m}_h > 0$, a contradiction. Therefore, $\bar{q} > 0$.

Since $(\bar{p}^0, \bar{q}) \gg 0$, the above system in turn implies that $\sum_h \bar{z}^0_h = \sum_h \bar{m}_h = 0$. Therefore, there exists a L-rational set expectations equilibrium. □

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[6] With a slight abuse of notation, we now let $z^0 = x^0 - e^0$ be the argument of the demand functions. To simplify notation further, we drop $\Phi_h(x^0, m)$ from the argument.

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