appendix to "Skill biased heterogeneous firms, trade liberalization, and the skill premium" by James Harrigan and Ariell Reshef, Canadian Journal of Economics.

APPENDIX A: THEORETICAL APPENDIX

This appendix proves Propositions 1 and 2 in the text. First we show that equilibrium is unique, and then we show that the movement from autarky to costly trade leads to an increase in the skill premium (in the skill-biased case) or no change in the skill premium (in the no-bias case). Throughout this appendix, we take the unskilled wage as our numeraire, so the skill premium s is the relative wage of skilled versus unskilled workers.

Finding equilibrium requires simultaneously solving the labor market equilibrium and free entry conditions for the equilibrium values of s and ϕ^*:

$$\Theta_1\left(s,\phi^*\right)=\frac{\iint\limits_{(\alpha,\varphi)\in D}\tilde{H}_{dv}g\left(\alpha,\varphi\right)d\alpha d\varphi+\tau^{1-\sigma}\iint\limits_{(\alpha,\varphi)\in X}\tilde{H}_{dv}g\left(\alpha,\varphi\right)d\alpha d\varphi}{\iint\limits_{(\alpha,\varphi)\in D}\tilde{L}_{dv}g\left(\alpha,\varphi\right)d\alpha d\varphi+\tau^{1-\sigma}\iint\limits_{(\alpha,\varphi)\in X}\tilde{L}_{dv}g\left(\alpha,\varphi\right)d\alpha d\varphi}=\frac{H}{L}\ , \tag{63}$$

$$\Theta_2\left(s,\phi^*\right)=f\iint\limits_{(\alpha,\varphi)\in D}\left[\left(\frac{\phi\left(\alpha,\varphi\right)}{\phi^*}\right)^{\sigma-1}-1\right]g\left(\alpha,\varphi\right)d\alpha d\varphi+$$

$$f_x\iint\limits_{(\alpha,\varphi)\in X}\left[\left(\frac{\phi\left(\alpha,\varphi\right)}{\phi_x^*}\right)^{\sigma-1}-1\right]g\left(\alpha,\varphi\right)d\alpha d\varphi=\delta f_e\ . \tag{64}$$

The cutoffs ϕ^* and ϕ_x^* define regions in (α,φ) space,

$$D\left(\phi^*,s\right)=\left\{(\alpha,\varphi)\in[0,1]\times\mathbb{R}_+^1:\phi^*\le\frac{\varphi}{s^\alpha}\right\}\ , \tag{65}$$

$$X\left(\phi_x^*,s\right)=\left\{(\alpha,\varphi)\in[0,1]\times\mathbb{R}_+^1:\phi_x^*\le\frac{\varphi}{s^\alpha}\right\}\ . \tag{66}$$

All firms with $(\alpha,\varphi)\in D$ are active in equilibrium while firms with $(\alpha,\varphi)\in X$ are also exporters, where $X\subset D$. These regions are illustrated in Figure 3.

A.1. Uniqueness

Our approach to uniqueness is to show that (63) and (64) define two curves in (s,ϕ^*) space, which we'll call the LME and FE schedules. Since these curves have opposite slopes, their intersection defines a unique solution. We show

$$LME:\frac{ds}{d\phi^*}=-\frac{\partial\Theta_1\left(s,\phi^*\right)}{\partial\phi^*}/\frac{\partial\Theta_1\left(s,\phi^*\right)}{\partial s}>0\ , \tag{67}$$

$$FE:\frac{ds}{d\phi^*}=-\frac{\partial\Theta_2\left(s,\phi^*\right)}{\partial\phi^*}/\frac{\partial\Theta_2\left(s,\phi^*\right)}{\partial s}<0\ . \tag{68}$$

A.1.1. The slope of the FE schedule

Using

$$\phi_x^*=\phi^*\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}=\beta\phi^*\ , \tag{69}$$

where $\beta = \tau \left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}} > 1$ and re-arranging (64) gives

$$f \iint\limits_{(\alpha,\varphi)\in D} \left[(\phi^*)^{1-\sigma}\, \phi(\alpha,\varphi)^{\sigma-1} - 1\right] g(\alpha,\varphi)\, d\varphi d\alpha +$$

$$f_x \iint\limits_{(\alpha,\varphi)\in X} \left[(\beta\phi^*)^{1-\sigma}\, \phi(\alpha,\varphi)^{\sigma-1} - 1\right] g(\alpha,\varphi)\, d\varphi d\alpha = \delta f_e \ . \tag{70}$$

We differentiate the integrals $I_D = \iint\limits_{(\alpha,\varphi)\in D} \left[(\phi^*)^{1-\sigma}\, \phi(\alpha,\varphi)^{\sigma-1} - 1\right] g(\alpha,\varphi)\, d\alpha d\varphi$ and

$I_X = \iint\limits_{(\alpha,\varphi)\in X} \left[(\beta\phi^*)^{1-\sigma}\, \phi(\alpha,\varphi)^{\sigma-1} - 1\right] g(\alpha,\varphi)\, d\alpha d\varphi$ with respect to $\phi^*$and s.

Differentiating I_D.

It is convenient to integrate first over φ, then over α. Writing out the limits of integration, and substituting $\phi(\alpha,\varphi)^{\sigma-1} = \varphi^{\sigma-1} s^{\alpha(1-\sigma)}$, I_D can re-written as

$$I_D = \int_0^1 \int_{s^\alpha \phi^*}^{\infty} (\phi^*)^{1-\sigma}\, \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha - \int_0^1 \int_{s^\alpha \phi^*}^{\infty} g(\alpha,\varphi)\, d\varphi d\alpha \ ,$$

or $I_D = I_D^1 + I_D^2$. Differentiating first with respect to s gives

$$\begin{aligned}
\frac{\partial I_D^1}{\partial s} &= \int_0^1 \int_{s^\alpha \phi^*}^{\infty} (\phi^*)^{1-\sigma}\, \varphi^{\sigma-1} \alpha(1-\sigma)\, s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi - \\
&\quad \int_0^1 \alpha s^{\alpha-1} \phi^* (\phi^*)^{1-\sigma} (s^\alpha \phi^*)^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha, s^\alpha \phi^*)\, d\alpha \\
&= (1-\sigma)(\phi^*)^{1-\sigma} \int_0^1 \int_{s^\alpha \phi^*}^{\infty} \varphi^{\sigma-1} \alpha s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi \\
&\quad -\phi^* \int_0^1 \alpha s^{\alpha-1} g(\alpha, s^\alpha \phi^*)\, d\alpha \ ,
\end{aligned}$$

$$\frac{\partial I_D^2}{\partial s} = -\left[0 - \int_0^1 \alpha s^{\alpha-1} \phi^* g(\alpha, s^\alpha \phi^*)\, d\alpha\right] = \phi^* \int_0^1 \alpha s^{\alpha-1} g(\alpha, s^\alpha \phi^*)\, d\alpha \ .$$

adding the pieces together gives

$$\frac{\partial I_D}{\partial s} = (1-\sigma)(\phi^*)^{1-\sigma} \int_0^1 \int_{s^\alpha \phi^*}^{\infty} \varphi^{\sigma-1} \alpha s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi \ .$$

Differentiating next with respect to ϕ^* gives

$$\begin{aligned}
\frac{\partial I_D^1}{\partial \phi^*} &= \int\limits_0^1 \int\limits_{s^\alpha \phi^*}^\infty (1-\sigma)(\phi^*)^{-\sigma} \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha \\
&\quad - \int\limits_0^1 s^\alpha (\phi^*)^{1-\sigma} (s^\alpha \phi^*)^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha, s^\alpha \phi^*)\, d\alpha \\
&= (1-\sigma)(\phi^*)^{-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \phi^*}^\infty \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha - \int\limits_0^1 s^\alpha g(\alpha, s^\alpha \phi^*)\, d\alpha \ ,
\end{aligned}$$

$$\frac{\partial I_D^2}{\partial \phi^*} = -\left[0 - \int\limits_0^1 s^\alpha g(\alpha, s^\alpha \phi^*)\, d\alpha\right] = \int\limits_0^1 s^\alpha g(\alpha, s^\alpha \phi^*)\, d\alpha \ .$$

adding the pieces together gives

$$\frac{\partial I_D}{\partial \phi^*} = (1-\sigma)(\phi^*)^{-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \phi^*}^\infty \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha \ .$$

Differentiating I_X.

$I_X(\phi^*, s)$ differs from $I_D(\phi^*, s)$ only in the lower limit of integration over φ. We define

$$I_X = \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^\infty (\beta\phi^*)^{1-\sigma} \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha - \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^\infty g(\alpha,\varphi)\, d\varphi d\alpha \ .$$

Calculations very similar to those just above establish

$$\frac{\partial I_X}{\partial s} = (1-\sigma)(\beta\phi^*)^{1-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^\infty \varphi^{\sigma-1} \alpha s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi \ ,$$

$$\frac{\partial I_X}{\partial \phi^*} = (1-\sigma)(\phi^*)^{-\sigma} \beta^{1-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^\infty \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha \ .$$

Summarizing the derivatives of the FE schedule.

Putting the pieces of the total derivative together,

$$\begin{aligned}
\frac{\partial \Theta_2}{\partial \phi^*} &= f\frac{\partial I_D}{\partial \phi^*} + f_x \frac{\partial I_X}{\partial \phi^*} \\
&= (1-\sigma)(\phi^*)^{-\sigma} \left[\begin{array}{c} f \int\limits_0^1 \int\limits_{s^\alpha \phi^*}^\infty \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha \\ + f_x \beta^{1-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^\infty \varphi^{\sigma-1} s^{\alpha(1-\sigma)} g(\alpha,\varphi)\, d\varphi d\alpha \end{array} \right] < 0 \ ,
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial \Theta_2}{\partial s} &= \frac{f\partial I_D}{\partial s} + f_x \frac{\partial I_X}{\partial s} \\
&= (1-\sigma)(\phi^*)^{1-\sigma} \left[\begin{array}{c} f \int\limits_0^1 \int\limits_{s^\alpha \phi^*}^{\infty} \varphi^{\sigma-1} \alpha s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi \\ + f_x \beta^{1-\sigma} \int\limits_0^1 \int\limits_{s^\alpha \beta\phi^*}^{\infty} \varphi^{\sigma-1} \alpha s^{\alpha(1-\sigma)-1} g(\alpha,\varphi)\, d\alpha d\varphi \end{array} \right] < 0 \ .
\end{aligned}
$$

The terms in brackets are strictly positive, while $(1-\sigma) < 0$, so both derivatives are strictly negative. We have thus confirmed the slope of the FE schedule given by (68).

A.1.2. The slope of the LME schedule

A direct calculus approach to establishing the slope of the LME schedule (63) is infeasible, so we proceed heuristically. We begin by re-writing the left hand side of (63) as an unskilled labor weighted average of each active firms skill intensity. The definitions of $\tilde{H}_{dv}$ and $\tilde{L}_{dv}$ that appear in (63) are

$$
\tilde{H}_{dv}(\alpha,\varphi,s) = \alpha s^{(1-\sigma)\alpha-1} \varphi^{\sigma-1} \ , \tag{71}
$$

$$
\tilde{L}_{dv}(\alpha,\varphi,s) = (1-\alpha) s^{(1-\sigma)\alpha} \varphi^{\sigma-1} \ . \tag{72}
$$

Dividing (71) by (72) gives

$$
\widetilde{hl}(\alpha,s) = \frac{\alpha}{1-\alpha} s^{-1} \ . \tag{73}
$$

Define the numerator and denominator on the left hand side of (63) as $\widetilde{H}_v(s,\phi^*)$ and $\widetilde{L}_v(s,\phi^*)$ respectively, so that we have

$$
\begin{aligned}
H_v &= M\rho^\sigma R P^{\sigma-1} \times \widetilde{H}_v(s,\phi^*) \ , \\
L_v &= M\rho^\sigma R P^{\sigma-1} \times \widetilde{L}_v(s,\phi^*) \ .
\end{aligned}
$$

and the following holds in equilibrium,

$$
\frac{H_v(s,\phi^*)}{L_v(s,\phi^*)} = \frac{\widetilde{H}_v(s,\phi^*)}{\widetilde{L}_v(s,\phi^*)} = \frac{H}{L} \ .
$$

Using the tautology $\widetilde{hl} \times \tilde{L}_{dv} = \tilde{H}_{dv}$, the definition of $\widetilde{L}_v(s,\phi^*)$, defining $\theta(\alpha,\varphi,s,\phi^*) = \tilde{L}_{dv}(\alpha,\varphi,s)/\widetilde{L}_v(s,\phi^*)$ and substituting, we re-write (63) as

$$
\begin{aligned}
\frac{H}{L} &= \iint\limits_{(\alpha,\varphi)\in D} \widetilde{hl}(\alpha,s)\,\theta(\alpha,\varphi,s,\phi^*)\, g(\alpha,\varphi)\, d\alpha d\varphi + \\
&\qquad \tau^{1-\sigma} \iint\limits_{(\alpha,\varphi)\in X} \widetilde{hl}(\alpha,s)\,\theta(\alpha,\varphi,s,\phi^*)\, g(\alpha,\varphi)\, d\alpha d\varphi \ .
\end{aligned} \tag{74}
$$

The interpretation of $\theta(\alpha,\varphi,s,\phi^*)$ is the share of unskilled labor employed by firms characterized by (α,φ) at the aggregate values (s,ϕ^*). By the definition of $\widetilde{L}_v(s,\phi^*)$,

$$
\iint\limits_{(\alpha,\varphi)\in D} \theta(\alpha,\varphi,s,\phi^*)\, g(\alpha,\varphi)\, d\varphi d\alpha + \tau^{1-\sigma} \iint\limits_{(\alpha,\varphi)\in X} \theta(\alpha,\varphi,s,\phi^*)\, g(\alpha,\varphi)\, d\varphi d\alpha = 1 \ .
$$

For firms that export, their total unskilled labor share is $\theta(\alpha,\varphi,s,\phi^*) \times \left(1+\tau^{1-\sigma}\right)$. Equation (74) is useful because it shows that the aggregate skill ratio is a weighted average of the firm-level skill ratios.

Now consider an incremental increase in the cutoff ϕ^*. By definition, this will lead to exit of the highest cost firms, with their weight in relative skill demand going to zero. By the assumption that technology is skill-biased, these firms are less skill-intensive than the firms that do not exit, causing an incipient relative excess demand for skilled labor. Thus to maintain relative labor market equilibrium, the skill premium s must rise when ϕ^* rises. Thus we conclude that the LME schedule is upward sloping in (s, ϕ^*) space. This concludes the demonstration that equilibrium is unique.

A.2. Proof of Proposition 1

Our proof of Proposition 1 proceeds by analyzing shifts in the LME and FE curves in the movement from autarky to costly trade. Since both curves shift up, the equilibrium skill premium must rise (see Figure A1).

A.2.1. Opening to trade causes shift up in FE curve

Consider the free entry condition (64). Under autarky, $X = \varnothing$ so (64) reduces to

$$\Theta_2(s, \phi^*) = f \int_0^1 \int_{s^\alpha \phi^*}^{\infty} \left[\left(\frac{\phi(\alpha, \varphi)}{\phi^*} \right)^{\sigma - 1} - 1 \right] g(\alpha, \varphi)\, d\alpha d\varphi = \delta f_e \ ,$$

where we have written out the limits of integration corresponding to the set of active firms D. Since the right hand side of (64) does not change in the move from autarky to costly trade, while we add a strictly positive integral $f_x \iint_{(\alpha,\varphi)\in X} \left[\left(\frac{\phi(\alpha,\varphi)}{\phi_x^*} \right)^{\sigma-1} - 1 \right] g(\alpha, \varphi)\, d\alpha d\varphi$, the first integral in (64) must get smaller. Holding ϕ^* fixed, inspection of the limits of integration confirms that this requires an increase in s, which corresponds to shift up of the FE curve (see Figure A1).

A.2.2. Opening to trade causes shift up in LME curve

In autarky, (74) reduces to

$$\int_0^1 \int_{s^\alpha \phi^*}^{\infty} \widetilde{hl}(\alpha, s)\, \theta(\alpha, \varphi, s, \phi^*)\, g(\alpha, \varphi)\, d\varphi d\alpha = \frac{H}{L} \ . \tag{75}$$

At the autarky equilibrium values of s and ϕ^*, consider an opening to costly trade. Two effects are immediate. First, because of fixed and variable export costs, only the most competitive firms will export, increasing their labor demand weights by the factor $\left(1 + \tau^{1-\sigma}\right)$ relative to the weights of non-exporters. Second, because of skill bias, newly exporting firms are more skill intensive on average than non-exporters. As a consequence of these two effects, relative skill demand increases when costly trade opens up. At the autarky equilibrium cutoff ϕ^*, s must increase to satisfy (74), which corresponds to an upward shift in the LME curve.

A.3. Proof of Proposition 2

When there is no skill bias, we can write the joint density as $g(\alpha, \varphi) = g_\alpha(\alpha)\, g_\varphi(\varphi)$. The marginal distributions are assumed to be uniform on $[0, 1]$ and Pareto on $[1, \infty)$ respectively

$$g_\alpha(\alpha) = 1, \qquad g_\varphi(\varphi) = k\ \varphi^{-(k+1)} \ .$$

Using these functional forms, the integrals in the numerator and denominator of (63) can be computed. Assuming $k > \sigma - 1$, and defining the parameter collection

$$\Delta = \frac{(\phi^*)^{\sigma-1-k}\left(s^k - 1 - k\log s\right)}{k\,(k-\sigma+1)\,[\log s]^2} \ ,$$

the integrals are

$$\iint\limits_{(\alpha,\varphi)\in D} \tilde{H}_{dv} g(\alpha,\varphi)\, d\varphi d\alpha = \int_0^1 \int_{s^\alpha \phi^*}^{\infty} \alpha s^{\alpha(1-\sigma)-1} \varphi^{\sigma-1} k\ \varphi^{-(k+1)} d\varphi d\alpha = s^{-1-k}\Delta \ ,$$

$$\tau^{1-\sigma} \iint\limits_{(\alpha,\varphi)\in X} \tilde{H}_{dv} g(\alpha,\varphi)\, d\varphi d\alpha = \tau^{1-\sigma}\beta^{\sigma-1-k} s^{-1-k}\Delta \ ,$$

$$\iint\limits_{(\alpha,\varphi)\in D} \tilde{L}_{dv} g(\alpha,\varphi)\, d\varphi d\alpha = \int_0^1 \int_{s^\alpha \phi^*}^{\infty} (1-\alpha)\, s^{\alpha(1-\sigma)} \varphi^{\sigma-1} k\ \varphi^{-(k+1)} d\varphi d\alpha = \Delta \ ,$$

$$\tau^{1-\sigma} \iint\limits_{(\alpha,\varphi)\in X} \tilde{L}_{dv} g(\alpha,\varphi)\, d\varphi d\alpha = \tau^{1-\sigma}\beta^{\sigma-1-k}\Delta \ .$$

Substituting and simplifying, (63) evaluates to

$$\Theta_1(s) = \frac{s^k - 1 - k\log s}{s + s^{k+1}(k\log s - 1)} = \frac{H}{L} \ . \tag{76}$$

Equation (76) is a single equation in the single unknown s. Differentiation establishes

$$\frac{d\Theta_1}{ds} = \frac{(1-k)\left(1-s^k\right)^2 + k\log s\left(1 - s^{2k} + k(k+1)\, s^k \log s\right)}{\left[s + s^{k+1}(k\log s - 1)\right]^2} < 0 \ ,$$

where the inequality holds for any $s > 1$. Thus, the solution s of (76) is unique, and monotonically decreasing in H/L. By inspection, the solution does not depend on any trade cost parameters, which proves that the skill premium is unaffected by opening to trade or trade liberalization in this case. This completes the proof of Proposition 2. A final note is that since $\lim_{s\to 1}\Theta_1(s) = 1$, (76) implies $s > 1$ if and only if $H/L < 1$. If $H/L \geq 1, s = 1$ by our assumption that skilled workers can perform unskilled jobs but not vice versa.

APPENDIX B: TECHNICAL APPENDIX

B.1. Applying the Plackett copula

We explain here how we simulate the primitive joint distribution $G(\alpha,\varphi)$. We take as given the parameters of the marginal distributions $\alpha \sim Beta(a,b)$ and $\varphi \sim Pareto(m,k)$ (m is the cutoff and k is the shape parameter) and the Plackett association parameter θ. First we generate draws from a joint uniform $[0,1]$ distribution following Nelsen (2006) (exercise 3.38 on page 99):

1. Draw two independent vectors of length I from a uniform $[0,1]$ distribution, U and X.

2. Set

$$\begin{aligned}
a_i &= X_i\left(1-X_i\right) \\
b_i &= \theta + a_i\left(\theta-1\right)^2 \\
c_i &= 2a_i\left(U_i\theta^2+1-U_i\right)+\theta\left(1-2a_i\right) \\
d_i &= \sqrt{\theta}\cdot\sqrt{\theta+4a_iU_i\left(1-U_i\right)\left(1-\theta\right)^2}\ .
\end{aligned}$$

3. Set $V_i = \left[c_i - \left(1-2X_i\right)d_i\right]/2b_i$ for $i = 1, 2, ...I$.

The vector V is distributed uniform $[0, 1]$. The pair (U, V) has the joint distribution function $C_\theta(u, v)$, where $C_\theta(u, v)$ is the Plackett copula. The correlation between U and V is governed by θ; it is positive for $\theta > 1$. One can think of U and V as marginal distribution functions. Using (U, V) we obtain α and φ by the inverses of the marginal distribution functions: $\alpha = G_\alpha^{-1}(U)$ and $\varphi = G_\varphi^{-1}(V)$. The pair (α, φ) follows the primitive joint distribution $G(\alpha, \varphi) = C_\theta\left(G_\alpha(\alpha), G_\varphi(\varphi)\right)$.

B.2. Solution for the symmetric open economy equilibrium

We take all the parameters, as well as the primitive distribution $G(\alpha, \varphi)$, as given. We solve for three endogenous variables: ϕ^*, w and s. All other endogenous variables and aggregates are functions of those variables. We solve for both nominal wages using gold as the numeraire. This makes the search for the equilibrium more efficient and robust. In addition, using gold as numeraire makes interpretation of changes of nominal values straightforward.

Draw I firms (technologies) from $G(\alpha, \varphi)$, denoted $\{(\alpha, \varphi)_i\}_{i=1}^I$. This set is fixed throughout the search for the equilibrium.

1. Guess initial values (ϕ_0^*, w_0, s_0) and set $\phi_x^* = \phi_0^*\tau\left(f_x/f\right)^{\frac{1}{\sigma-1}}$.

2. Set $\phi_i = \varphi_i/(s_0^{\alpha_i}w_0^{1-\alpha_i})$ for all i. Collect surviving firms such that $\phi_i > \phi_0^*$; this leaves us with J active firms: $\{\phi_j\}_{j=1}^J$ and the commensurate $\{(\alpha, \varphi)_j\}_{j=1}^J$. Collect exporters such that $\phi_i > \phi_x^*$; this leaves us with T exporters: $\{\phi_t\}_{t=1}^T$ and the commensurate $\{(\alpha, \varphi)_t\}_{t=1}^T$. Finally, set $\chi_d = J/I$ (probability of entry) and $\chi = T/J$ (export probability conditional on entry).

3. Compute three deviations from equilibrium relationships

$$\begin{aligned}
\Delta_{fe} &= f\frac{1}{J}\sum_{j=1}^{J}\left[\left(\phi_j/\phi_0^*\right)^{\sigma-1}-1\right]+f_x\chi\frac{1}{T}\sum_{t=1}^{T}\left[\left(\phi_t/\phi_x^*\right)^{\sigma-1}-1\right]-\delta f_e/\chi_d \\
\Delta_{rle} &= \frac{\frac{1}{J}\sum_{j=1}^{J}\alpha_j\phi_j^{\sigma-1}+\chi\tau^{1-\sigma}\frac{1}{T}\sum_{t=1}^{T}\alpha_t\phi_t^{\sigma-1}}{\frac{1}{J}\sum_{j=1}^{J}\left(1-\alpha_j\right)\phi_j^{\sigma-1}+\chi\tau^{1-\sigma}\frac{1}{T}\sum_{t=1}^{T}\left(1-\alpha_t\right)\phi_t^{\sigma-1}}\cdot\frac{w_0}{s_0}-\frac{H}{L} \\
\Delta_{gold} &= Hs+Lw-G/2\ ,
\end{aligned}$$

where G is the amount of gold in the world.

Equilibrium is found when all Δ are all equal to zero. We search for (ϕ_0^*, w_0, s_0) such that these conditions are met. We use the numerical solver `fsolve` in Matlab to do this.

The numerical equilibrium relationships are written differently from those in the main text to reflect the computation methodology; however, they are the same as in the main text. All averages are approximations of means, and take into account the truncations and correct distribution functions. For example,

$$\frac{1}{J}\sum_{j=1}^{J}\left[\left(\phi_j/\phi_0^*\right)^{\sigma-1}-1\right]$$

approximates

$$\iint\limits_{(\alpha,\varphi)\in D} \left[\left(\frac{\phi(\alpha,\varphi)}{\phi^*} \right)^{\sigma-1} - 1 \right] \frac{g(\alpha,\varphi)}{\chi_d} d\alpha d\varphi \ ,$$

not

$$\iint\limits_{(\alpha,\varphi)\in D} \left[\left(\frac{\phi(\alpha,\varphi)}{\phi^*} \right)^{\sigma-1} - 1 \right] g(\alpha,\varphi)\, d\alpha d\varphi \ ,$$

because $\{\phi_j\}_{j=1}^J$ and the commensurate $\{(\alpha,\varphi)_j\}_{j=1}^J$ are both in D by construction and are both distributed according to $g(\alpha,\varphi)/\chi_d$. So while the free entry condition and relative labor equilibrium equation in the main text do not involve χ_d or χ, here we must correct the means by the relevant probabilities to match these relationships in the text.

B.3. Solution for the autarky equilibrium

The solution of the model for an economy in autarky is very similar to the solution for the symmetric open economy case with one difference: we do not have a set of exporters. We solve for three endogenous variables: ϕ^*, w and s. All other endogenous variables and aggregates are functions of those variables. The three deviations from autarky equilibrium relationships are

$$\begin{aligned}
\Delta_{fe} &= f\frac{1}{J}\sum_{j=1}^{J}\left[\left(\phi_j/\phi_0^*\right)^{\sigma-1} - 1\right] - \delta f_e/\chi_d \\
\Delta_{rle} &= \frac{\frac{1}{J}\sum_{j=1}^{J}\alpha_j\phi_j^{\sigma-1}}{\frac{1}{J}\sum_{j=1}^{J}\left(1-\alpha_j\right)\phi_j^{\sigma-1}} \cdot \frac{w_0}{s_0} - \frac{H}{L} \\
\Delta_{gold} &= Hs + Lw - G \ .
\end{aligned}$$

Equilibrium is found when all Δ are all equal to zero. We search for (ϕ_0^*, w_0, s_0) such that these conditions are met. We use the numerical solver `fsolve` in Matlab to do this.

B.4. Solution for the equilibrium with differences in factor endowments

B.4.1. Mathematical details

Solving for the equilibrium with differences in factor endowments involves all endogenous variables, including aggregates, simultaneously. However, it is possible to compartmentalize the equilibrium as follows. Define the following vector of seven equilibrium variables

$$\mu' = \left(s^A, w^B, s^B, \phi^{*B}, \phi^{*B}, P^A, P^B \right) \ ,$$

where we set w^A as numeraire. The remainder of the equilibrium values are given by

$$\eta' = \left(R^A, R^B, \phi_x^{*A}, \phi_x^{*B}, \chi_d^A, \chi_d^B, \chi^A, \chi^B, \tilde{\phi}^A, \tilde{\phi}^B, \tilde{\phi}_x^A, \tilde{\phi}_x^B, M^A, M^B \right) \ .$$

The entire equilibrium is determined by a system of 21 equations, partitioned as follows

$$F(\mu,\eta) = \begin{bmatrix} f(\mu,\eta) \\ g(\mu,\eta) \end{bmatrix} = 0 \ .$$

f involves three factor market clearing conditions (equations (55) and (56) for each country, with one equation discarded as redundant), two free entry conditions (equation (27) for each country), and price indices (57) and (58). g involves aggregate revenue equations (42), the relationships between entry and exporting cutoffs (53) and (54), probability equations (21) and (22), average competitiveness (23) and (24), and firm mass equations (44).

Consider $g(\mu,\eta)=0$. By the implicit function theorem, there exists a function $\eta=\eta(\mu)$ such that $g[\mu,\eta(\mu)]=0$. The function $\eta(\mu)$ exists because (a) g is continuously differentiable; and (b) given the particular partition we chose, the Jacobian matrix $[\partial g/\partial\eta']$ is nonsingular for all admissible values of η. We use $\eta(\mu)$ in

$$F[\mu,\eta(\mu)]=0$$

to find values of μ that satisfy all equilibrium conditions.

B.4.2. Numerical solution

We take all the parameters, as well as the primitive distribution $G(\alpha,\varphi)$, as given. We solve for 8 endogenous variables: s_0^A, w_0^A, s_0^B, w_0^B, ϕ_0^{*A}, ϕ_0^{*B}, P_0^A and P_0^B. All other endogenous variables and aggregates are functions of those variables. We solve for all nominal variables using gold as the numeraire. This makes the search for the equilibrium more efficient and robust. In addition, using gold as numeraire makes interpretation of changes of nominal values straightforward.

Draw I firms (technologies) from $G(\alpha,\varphi)$, denoted $\{(\alpha,\varphi)_i\}_{i=1}^{I}$. This set will not change throughout the search for the equilibrium.

1. Guess initial values
$$\mu_0'=\left(s_0^A,w_0^A,s_0^B,w_0^B,\phi_0^{*A},\phi_0^{*B},P_0^A,P_0^B\right)$$
and set
$$\begin{aligned}R^c &= H^c s_0^c+L^c w_0^c\\ \phi_x^{*c} &= \phi_0^{*c}\tau\left(\frac{P_0^c}{P_0^{c'}}\right)\left(\frac{R^c}{R^{c'}}\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\end{aligned}\quad .$$
for each country $c\in\{A,B\}$ and $c'=\{A,B\}\backslash c$.

2. Set $\phi_i^c=\varphi_i/((s_0^c)^{\alpha_i}(w_0^c)^{1-\alpha_i})$ for all i. Collect surviving firms such that $\phi_i^c>\phi_0^{*c}$; this leaves us with J^c active firms: $\{\phi_j^c\}_{j=1}^{J^c}$ and the commensurate $\left\{(\alpha,\varphi)_j\right\}_{j=1}^{J^c}$. Collect exporters such that $\phi_i^c>\phi_x^{*c}$; this leaves us with T^c exporters: $\{\phi_t^c\}_{t=1}^{T^c}$ and the commensurate $\{(\alpha,\varphi)_t\}_{t=1}^{T^c}$. Note that ϕ_i^c are indexed by country because the wages are different.

3. Compute the following auxiliary objects:
$$\begin{aligned}\chi^c &= T^c/J^c\\ \chi_d^c &= J^c/I\\ M^c &= \frac{N}{\sigma\left(\frac{\delta}{\chi_d^c}f_e+f+\chi^c f_x\right)}\\ \widetilde{\phi}^c &= \left[\frac{1}{J^c}\sum_{j=1}^{J^c}(\phi_j^c)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}\\ \widetilde{\phi}_x^c &= \left[\frac{1}{T^c}\sum_{t=1}^{T^c}(\phi_t^c)^{\sigma-1}\right]^{\frac{1}{\sigma-1}}\\ \Upsilon^c &= \tau^{1-\sigma}\left(\frac{P^{c'}}{P^c}\right)^{\sigma-1}\left(\frac{R^{c'}}{R^c}\right)\end{aligned}\quad .$$

4. For each country c compute four deviations from equilibrium relationships

$$\begin{aligned}
\Delta_{fe}^{c} &= f\frac{1}{J^{c}}\sum_{j=1}^{J^{c}}\left[\left(\phi_j/\phi_0^{*c}\right)^{\sigma-1}-1\right]+f_x\chi^{c}\frac{1}{T^{c}}\sum_{t=1}^{T^{c}}\left[\left(\phi_t^{c}/\phi_x^{*}\right)^{\sigma-1}-1\right]-\delta f_e/\chi_d^{c} \\
\Delta_{p}^{c} &= \left[M^{c}\left(\rho\widetilde{\phi}^{c}\right)^{\sigma-1}+\chi^{c'}M^{c'}\tau^{1-\sigma}\left(\rho\widetilde{\phi}_x^{B}\right)^{\sigma-1}\right]^{\frac{1}{1-\sigma}}-P_0^{A} \\
\Delta_{h}^{c} &= M^{c}\rho^{\sigma}\left(P_0^{c}\right)^{\sigma-1}R^{c}\left[\frac{1}{J^{c}}\sum_{j=1}^{J^{c}}\alpha_j\left(\phi_j^{c}\right)^{\sigma-1}+\chi^{c}\Upsilon^{c}\frac{1}{T^{c}}\sum_{t=1}^{T^{c}}\alpha_t\left(\phi_t^{c}\right)^{\sigma-1}\right]/s_0^{c}-H^{c} \\
\Delta_{l}^{c} &= M^{c}\rho^{\sigma}\left(P_0^{c}\right)^{\sigma-1}R^{c}\left[\frac{1}{J^{c}}\sum_{j=1}^{J^{c}}\left(1-\alpha_j\right)\left(\phi_j^{c}\right)^{\sigma-1}+\chi^{c}\Upsilon^{c}\frac{1}{T^{c}}\sum_{t=1}^{T^{c}}\left(1-\alpha_t\right)\left(\phi_t^{c}\right)^{\sigma-1}\right]/w_0^{c}-L^{c}\ .
\end{aligned}$$

In addition, compute the deviation from global nominal output,

$$\Delta_{gold}=R^{A}+R^{B}-G\ ,$$

where G is the amount of gold in the world. We do not use Δ_l^{B}, so that the system is identified exactly.

Equilibrium is found when all Δ are all equal to zero. We search for μ_0 such that these conditions are met. We use the numerical solver `fsolve` in Matlab to do this.

Some of the numerical equilibrium relationships are written differently from those in the main text to reflect the computation methodology; however, they are the same as in the main text. All averages are approximations of means, and take into account the truncations and correct distribution functions; see Section B.2.